equidistant from z', so z' is not on the boundary of the locus of z. Similarly we may show that z_1' and z_2' cannot be the points of C_1 and C_2 nearest to z', so z_1' and z_2' must satisfy the conditions stated.

The points z', z_1', A_1 are collinear and similarly the points z', z_2', A_2. The point A_1 is on the segment $z'z_1'$ if and only if A_2 is not on the segment $z'z_2'$. The distances $z'z_1'$ and $z'z_2'$ are equal by hypothesis, so the distances $z'A_1$ and $z'A_2$ differ by the sum of the distances A_1z_1' and A_2z_2', that is, by the sum of the radii of C_1 and C_2. Then z' lies on the hyperbola whose foci are A_1 and A_2 and whose "constant difference" is the sum of the radii of C_1 and C_2. The locus of z is not the entire plane and therefore has a boundary; the locus contains the perpendicular bisector of A_1A_2 but no point of the line A_1A_2 not on the finite segment A_1A_2. Hence this locus must be bounded by the entire hyperbola and is the exterior of the hyperbola. This completes the proof of the theorem.

Denote by B_1' and B_1'' and by B_2' and B_2'' the intersections of the line A_1A_2 with C_1 and C_2, respectively, determined so that B_1' separates B_1'' and B_2'' but B_2' does not separate B_1'' and B_2''. The hyperbola cuts the line A_1A_2 at the midpoints of the segments $B_1'B_2'$ and $B_1''B_2''$. The reader will easily prove that the asymptotes of the hyperbola are the perpendicular bisectors of the transverse tangents to C_1 and C_2.

For the problem just considered, the point z' can never lie on the segment $z_1'A_1$; otherwise z_2' would be within C_1, and C_1 and C_2 are supposed to be mutually external. But if we modify our problem by assigning to z_1 as locus the region of the plane *exterior to* C_1, and if the locus of z is not the entire plane, the point z' always lies between A_1 and z_1'. The locus of z can be shown to be the region of the plane exterior to a certain ellipse whose foci are A_1 and A_2.

If we modify our problem by assigning to z_1 as locus a *half plane*, while the locus of z_2 remains the interior of C_2, the locus of z is either the entire plane or the exterior of a parabola whose focus is A_2 and whose directrix is parallel to the boundary of the locus of z_1.

AMONG MY AUTOGRAPHS.

By DAVID EUGENE SMITH, Columbia University.

20. Babbage Visits Mme. Laplace.

Sir John Herschel, speaking of the status of mathematics and astronomy in Great Britain at the opening of the nineteenth century, remarked that "Mathematics were at the last gasp, and Astronomy nearly so." It was for this reason that he, in conjunction with George Peacock and Charles Babbage, formed the so-called "Analytical Society", the purpose of which was to introduce into Cambridge the Continental type of the calculus and, in general, to revivify the mathematics of England. The same three scholars were influential in establishing the Astronomical Society of London, and each was among the leaders in other efforts of a similar nature, one of Babbage's most important papers being entitled "Reflections on the Decline of Science in England" (1830).

Although we commonly think of Babbage with relation to his calculating engine (part of which, curiously enough, found its way to the Dudley Observatory at Albany), he held the chair of Lucasian professor of mathematics at Cambridge, and contributed worthily to the science of astronomy, to higher algebra, and to physics.

In 1840 he was in Paris, and M. Alexis Bouvard (1767–1843) took him to see the widow of Laplace. Bouvard, who became connected as astronomer with the national observatory at Paris in 1793, had assisted Laplace in his computations for the *Mécanique Céleste,* and was therefore in friendly relations to the latter's household. Bouvard was then seventy-three years old, while Babbage was only about fifty.

As Babbage was leaving Paris he wrote to M. Bouvard a letter of appreciation and thanks, acknowledging the gift of a portrait of Laplace which had long been in Bouvard's study. The letter, now in my collection, shows something of the personal side of Babbage and his natural courtesy on such an occasion, and is as follows:

(M. Alexis Bouvard.)
À Paris

My dear Sir:

I cannot leave Paris without thanking you for the very delightful day I spent with you in the society of Madame de Laplace. I shall preserve with the respect it deserves the portrait of her illustrious husband and if any circumstance could have rendered the gift more valuable it is the fact that it has so long adorned the study of that chosen friend by whose unremitted labors the Mecanique Coeleste received such valuable assistance.

I am My Dear Sir
with the greatest respect and Regard
Ever very sincerely Yours,
C. Babbage

Paris 2 Sep. 1840

P.S. I enclose two copies of the drawing of the Engine of Differences on [*sic*] to replace your own and one which I beg your nephew to do me the favour to accept.

21. De Morgan and the Libri Controversy.

If ever there was an eccentric genius in mathematics, and one who scattered his energies so recklessly as to render notable success in any one line impossible, that man was Augustus De Morgan (1806–1871). He did a great deal of good by means of his articles in the *Penny Cyclopedia,* in the *British Almanac and Companion,* and in Smith's *Dictionary of Greek and Roman Biography* (with some curious errors), and also by means of his works on the calculus, logic, double algebra, insurance, and trigonometry, and he gave the mathematical world much food for pleasant thought in his *Budget of Paradoxes;* but at his best he was an eccentric man. He stood rigidly for principle, which the cynic might say was in itself an evidence of eccentricity, and it must be confessed that some of his acts seem to show that he was such a slave to his principles as to justify such a cynic's belief.

An evidence of this eccentricity, perhaps excusable under the particular circumstances, may be seen in the letter which is given below, and which is now

among my autographs. There is nothing to show to whom it was written, but it refers to one phase of the famous Libri controversy which aroused great interest and very bitter feeling at the beginning of the second half of the nineteenth century. Libri, the well-known historian of mathematics, had been accused of stealing many of the books that made up his extensive library. The French courts had, in his absence, convicted him of the crime. He had fled to London and had become an intimate friend of the De Morgans, and this letter shows the faith that De Morgan himself had in the innocence of the accused. It also shows the esteem in which De Morgan held Chasles, the French geometer, and the lengths to which a question of principle could carry him in what was, as regards the scientists of the time, an international dispute.

The letter, which has not before been published to my knowledge, is as follows:

7 Camden Street
March 18/54

My dear Sir:

I am fully of opinion that M. Chasles has high claims on the Royal Society—But, though I am aware it is useless, I for one, will never give an opinion on his claims except in connexion with those of another, whose claims I hold still higher—I mean *Libri*. What I should say of Chasles as an historian, I should say of Libri in a higher degree, and both are truly original mathematicians. I am well aware that though the malignant accusations made against Libri have been refuted in a way in which charges very seldom are refuted—and though the evidence of determination to prevent his having a fair trial is as clear as any evidence can be—yet the power of ignoring notorious facts which bodies of men possess in their collective capacity,—and the disposition of our country to acknowledge all results of foreign law, however little it may agree with our own in fairness of procedure—would make it useless to propose Libri to the R.S. I myself should not entertain a sufficiently high opinion of any English Society to venture on such a step.

This being understood, and speaklg [*sic*, apparently for "speaking"] of Chasles absolutely—I should describe him as one of the very few mathem^{ns}—and the only one of French birth—who attend to the history of the science. In this subject he is a man of real learning—deep in original sources. His *Apercu* &c is but one of his contributions—and all of them throw new light on the older history of geometry & arithmetic.

In mathematics—as you know—he is among the foremost of those who have developed the powers of geometry to that extent which would have left algebra behind as an instrument—if algebra itself had not received new developments in the mode of applying it to geometry.

Chasles, I have no doubt, has been underrated by his countrymen. In truth, who was there in France (except Libri) to appreciate him? But if ever history and geometry revive in that country—Chasles will be looked upon as the founder of a school.

Chasles should have been elected years ago—He is nоw in the Institute—You might have given his passport—you can now only *viser* it—This is the great fault of our medals, associateships &c—and springs partly from moral fear, partly from it being nobody's business to form an independ^t opinion on reputations.

Yours very truly,
A. De Morgan

QUESTIONS AND DISCUSSIONS.

Edited by C. F. Gummer, Queen's University, Kingston, Ont., Canada.

REPLIES.

Of the two replies to Question 44, that of Professor Gilman contains a solution of the problem proposed, in the shape of a simple formula for the co-factors in the given "binomial" determinant.